**QUESTION BANK**

**DEPARTMENT OF MATHEMATICS**

**COURSE : B.Tech (FOOD TECH)**

**SUBJECT : TRANSFORM AND DIFFERENTIAL EQUATIONS**

**COURSE CODE: UBMT302**

**SEMESTER: III**

**UNIT-I**

**PARTIAL DIFFERENTIAL EQUATIONS**

**PART-A**

1. Form the PDE by eliminating *a* and *b* from 
2. Eliminatethe *a* and *b* from *z = ax + by +ab*
3. Find the complete solution of *p* $-$ *q* = 1
4. Form the PDE by eliminating the arbitrary constants *a* and *b* from
5. Define: Order and Degree of partial differential equation.
6. Form the PDE by eliminating the arbitrary function
7. Eliminate the arbitrary function 
8. Form the PDE by eliminating the arbitrary function 
9. Solve: $\frac{∂^{2}z}{∂x∂y}=x$ + *y*
10. Eliminate the arbitrary function 
11. Solve: $\sqrt{p}$ + $\sqrt{q}$ = 1
12. Eliminate the arbitrary constants *a* and *b* from
13. Solve: $\frac{∂^{2}z}{∂x^{2}}= 0$
14. Eliminate the arbitrary constants *a* and *b* from 
15. Solve: *p + q =* 1
16. Solve : *p q +p + q =* 0
17. Find the Complete integral of 
18. Find the Complete integral of *pq* = 4
19. Find the Complete integral of *p* + *x* = *q y*
20. Find the Complete integral of 
21. Find the Complete integral of 
22. Define: Lagrange’s Linear Equations.
23. Form the PDE by eliminating the arbitrary constants *a* and *b* from 
24. Solve: *z = p + q*
25. Solve : *p + q = x + y*

**PART-B**

1. Form the PDE by eliminating a and b from $z=a\left(x+logy\right)-\frac{x^{2}}{2}-b$
2. Form the PDE by eliminating the arbitrary function 
3. Form the PDE by eliminating the arbitrary constants from 
4. Solve 
5. Solve 
6. Solve: 
7. Solve: 
8. Solve
9. Find the general solution of 
10. Find the Complete integral of 
11. Solve: 
12. Solve: 
13. Find the differential equation of all spheres of fixed radius having their centers in the *x y* plane.
14. Eliminate the arbitrary constants *a* and *b* from
15. Form the PDE by eliminating the arbitrary function

**PART-C**

1. Find the Singular solution of 
2. Find the Singular solution of 
3. Solve: also find Singular solution.
4. Find the Singular solution of .
5. Solve: (*mz*  *ny*) *p + q* (*nx*  *lz*) *= ly*  *mx*
6. Find the general solution of 
7. Solve: 
8. Solve: 
9. Find the general solution of 
10. Solve: 

 **UNIT- II**

**FOURIER SERIES**

**PART- A**

1. Find the Fourier constant $b\_{n}$ for $xsinx$ in (-$π,π)$.
2. Find the constant $a\_{0}$ of the Fourier series for the function $f\left(x\right)=x$ in $0\leq x\leq 2π.$
3. Write the Dirichlet’s conditions for Fourier series.
4. Define Fourier constants.
5. When the Fourier expansion contains only cosine terms or sine terms?
6. Define periodic function for Fourier series.
7. Find the Fourier constant $a\_{n}$ for $xcosx$ in (-$π,π)$.
8. Write the formula for Fourier constant for$ f(x)$ in the interval (-$π,π)$.
9. Write the formula for Fourier constant for$ f(x)$ in the interval (0$,2π)$.
10. Find the constant $a\_{0}$ of the Fourier series for the function $f\left(x\right)=x^{2}$ in $0\leq x\leq 2π.$
11. Find the Fourier constant $b\_{n}$ for $x^{2}$ in (-$π,π)$.
12. What do you meant by Harmonic Analysis?
13. Write $a\_{0},a\_{n}$ in the expression $x+x^{3}$as a Fourier series in (-$π,π)$.
14. Define Fourier Series
15. Write the formula for the half -Range Sine series in (0$,c)$.
16. Write the formula for the half -Range Cosine series in (0$,c)$.
17. What is the sum of the Fourier Series at a point $x=x\_{0} $where the function $f(x)$ has a finite discontinuity.
18. What is the value of the Fourier constant $a\_{n}$ when odd function $f(x)$ is expanded in

 (-$π,π$)

1. Find the constant $a\_{0}$ of the Fourier series for the function $f\left(x\right)=k $in $0\leq x\leq 2π.$
2. Write the formula of Fourier constant for $f\left(x\right) $in ($-l$ $, l)$.
3. If $f\left(x\right)$ is an odd function defined in ($-1$ $, 1)$ what are the values of $a\_{0} and a\_{n}$
4. Find the constant $a\_{0}$ of the Fourier series for the function $f\left(x\right)=x^{3}$ in $0\leq x\leq 2π.$
5. Write any two examples of periodic function for Fourier series.
6. Define Fourier Co-efficient.
7. Pick out the odd functions with solutions**:**$ sinx,x cosx,Cosx.$

**PART – B**

1. Obtain the Fourier series for $f\left(x\right)=e^{-x}$ in the interval $0\leq x\leq 2π.$
2. Express  as a Fourier series in the interval (-$π,π)$.
3. Show that the Fourier Series for $f\left(x\right)=x$ ,$ -π<x<π $given by
4. Find the Fourier constant $a\_{n}$ for $xsinx $in (-$π,π)$.
5. Find the Fourier constant $a\_{n}$ for $f\left(x\right)=\left|x\right|, -π<x<π.$
6. Find the Fourier series for the function define by .
7. Find the half-range cosine series for the function $f\left(x\right)=x^{2}$ in the range $0\leq x\leq π.$
8. Expressas a half-range sine series in 0 < x < 2.
9. Expressas a half-range cosine series in 0 < x < 2.
10. Find the Fourier constant $b\_{n}$ for $f\left(x\right)=\left(\frac{π-x}{2}\right)^{2}, 0<x<2π.$
11. The following table gives the variations of a periodic function over a period T.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | T/6 | T/3 | T/2 | 2T/3 | 5T/6 | T |
| *f* (*x*) | 1.98 | 1.3 | 1.05 | 1.3 | -0.88 | -0.25 | 1.98 |

Show that  where.

1. Find the half range sine series of in (0,1)
2. Find the half range sine series of in (0,1)
3. Obtain the half-range cosine series for  in 0 < x < $π.$
4. Find the half range sine series of in (0,$ π$)

**PART-C**

1. Expand $f\left(x\right)=xsinx$ as a Fourier series in (0,2$ π)$.
2. Prove that $x^{2}=\frac{π^{2}}{3}+4\sum\_{n=1}^{\infty }\left(-1\right)^{n}\frac{cosnx}{n^{2}}$,$ -π<x<π.$ Hence show that (i)$\sum\_{}^{}\frac{1}{n^{2}}=\frac{π^{2}}{6}.$

 (ii)$ \frac{1}{1^{2}}-\frac{1}{2^{2}}+\frac{1}{3^{2}}-\frac{1}{4^{2}}+…=\frac{π^{2}}{12}$ (iii)$\sum\_{}^{}\frac{1}{(2n-1)^{2}}=\frac{π^{2}}{8}$.

1. If $f\left(x\right)=\left(\frac{π-x}{2}\right)^{2}$ in the range 0 to 2$ π, show that f\left(x\right)= \frac{π^{2}}{12}+\sum\_{n=1}^{\infty }\frac{cosnx}{n^{2}}$.
2. Find the Fourier series expansion for $f\left(x\right)=-π, -π<x<0$

 $x, 0<x<π$ .

Deduce that $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+…=\frac{π^{2}}{8}$

1. Find the Fourier series to represent $x^{2}$ in the interval$(-l,l)$.
2. If$ f\left(x\right)=\left|cosx\right|$, expand $f\left(x\right)$ as a Fourier series in the interval (-$π,π)$.
3. Obtain a Fourier series for the function $f\left(x\right)=\left|x\right|, -π<x<π, $

And Deduce that$\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}}+\frac{1}{7^{2}}…=\frac{π^{2}}{8}$.

1. Find the Fourier series for the function define by 
2. Find the Fourier series expansion of period 2 for the function *y* = *f* (*x*) which is defined in (0,2) by means of the table of values given below .Find the series up to the **second** harmonic.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 |  |  |  |  |  |  |
| *y* | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

1. Find the Fourier series as far as second harmonic to represent the function given the following data.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 2 | 3 | 4 | 5 |
| *f* (*x*) | 9 | 18 | 24 | 28 | 26 | 20 |

**UNIT-III**

**APPLICATION OF PARTIAL DIFFERENTIAL EQUATIONS**

**PART A**

1. In wave equation $u\_{tt}-a^{2}u\_{xx}=0$ what does $a^{2}$ stands for?
2. State: One-dimensional wave equation.
3. State: Two dimensional heat flow equation for unsteady state.
4. State: the Laplace equation
5. State Fourier law of heat conduction.
6. State one dimensional heat-flow equation for unsteady state.
7. What are the possible solutions for one dimensional heat equation?
8. What are the possible solutions of Laplace equation?
9. What are the possible solutions of one dimensional wave equation?
10. What is basic difference between the solutions of one dimensional wave & heat flow equation?
11. Classify the partial differential equation $f\_{xx}=f\_{yy}$
12. Classify the partial differential equation $u\_{xx}-2u\_{xy}$+$u\_{yy}$= 0
13. Classify the partial differential equation $x u\_{xx}+yu\_{yy}$= 0, if *x* and *y* are positive.
14. Classify the partial differential equation $u\_{xx}-2u\_{xy=0}$
15. Classify the partial differential equation $u\_{xx}-2u\_{xy=0}$
16. Classify the partial differential equation $u\_{xx}+xu\_{xy=0}$
17. Classify the partial differential equation $y^{2}u\_{xx}$+$u\_{yy}+u\_{x}^{2}$ +$ u\_{y}^{2}+7=0$
18. What is the most general solution of one dimensional wave equation if the displacement of the string is given?
19. What is the most general solution of one dimensional heat equation if velocity of the string is given?
20. What is the most general solution of two dimensional heat equation?
21. Write any two solutions of the Laplace equation obtained by the method of separation of variable.
22. What are the assumptions are made to derive one dimensional heat equation?
23. What are the assumptions are made to derive one dimensional wave equation?
24. Define: Steady state temperature distribution.
25. Write the solution of one dimensional heat flow equation, when the time derivative is absent.

**PART – B**

1. Find the D'Alembert's solution of one dimensional wave equation.
2. Find the D'Alembert's solution of two dimensional steady heat equation.
3. A string of length  is fixed at both the ends x= 0 and x =.The mod point of the string is displaced transversely through a small distance ‘ *b*’. Find the displacement of string initially.
4. A string of length  is fixed at both the ends x = 0 and x =.The mod point of the string is displaced transversely through a small distance$ \frac{l}{3}$. Find the displacement of string initially.
5. The ends A & B of a rod of length 10 $cm$ long have their temperature kept 20$℃$ and 70$℃$ and the rod is reached steady state. Find the steady state temperature distribution on the rod.
6. An insulated rod of length $l$ has its ends A and B maintained at $0℃$ and100 $℃$ respectively and the rod is reached state condition. Find the temperature at any point in terms of its distance from one end.
7. A rod 30 cm long has its ends A and B kept at 20$℃$ and 80$℃$ respectively until steady state condition prevail .Find the steady state temperature in the rod.
8. An insulated rod of length $l=60cm$ has its ends at A and B maintained at 30$℃$ and

40$℃$ respectively. Find the steady state solution.

1. When the end of a rod length 20 cm are maintained at the temperature 10$℃$ and 20$℃$

respectively until steady state is prevailed .Determine the steady state temperature of the rod.

1. Solve using separation of variables method $yu\_{x}$+$ xu\_{y}$=0.
2. Using the method of separation of variables, solve $x^{2}q+y^{3}p=0$
3. The ends A & B of a rod of length 20 $cm$ long have their temperature kept 10$℃$ and 20$℃$. Find the steady state temperature distribution on the rod.
4. A rod *l* = 20 cm long has its ends A and B kept at 30$℃$ and 60$℃$ respectively until steady state condition prevail .Find the steady state temperature in the rod.
5. The ends A and B of a rod of length 20 $cm$ long have their temperature kept 20$℃$ and 80$℃$. Find the steady state temperature distribution on the rod.
6. When the end of a rod length 20 cm are maintained at the temperature 10$℃$ and 20$℃$

respectively until steady state is prevailed .Determine the steady state temperature of the rod.

**PART C**

1. Using the method of separation of variables, solve $\frac{∂u}{∂x}=2\frac{∂u}{∂t}+u$

where $u\left(x,0\right)=6e^{-3x}$

1. Using the method of separation of variables, solve $\frac{∂u}{∂x}=4\frac{∂u}{∂y}$,

Given that$ u\left(0,y\right)=8e^{-3y}$.

1. Using the method of separation of variables, solve$ x^{2}\frac{∂u}{∂x}+y^{2}\frac{∂u}{∂y}=0$.
2. A tightly stretched string of length $l$ with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity$v\_{0}sin\frac{πx}{l}$. Find the displacement $y\left(x,t\right).$
3. A tightly stretched string with fixed end points $x=0 $and $x=l$ is initially in a position given by$ y=y\_{0}sin^{3}\frac{πx}{l}$. If it is released from rest from this position, find the displacement $y\left(x,t\right).$
4. Using the method of separation of variables, solve$ 4\frac{∂u}{∂x}+\frac{∂u}{∂y}=3u,$ given

 $u=3e^{-y}-e^{-5y}$ when $x=0.$

1. A tightly stretched string with fixed end points $x=0$and $x=l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $λx\left(l-x\right),$ find the displacement of the string at any distance x from one end at any time t.
2. Solve completely the equation $\frac{∂^{2}y}{∂t^{2}}=c^{2}\frac{∂^{2}y}{∂x^{2}}$, representing the vibrations of a string of length $l$, fixed at both ends, given that $y\left(0,t\right)=0;y\left(l,t\right)=0;y\left(x,0\right)=f(x)$ and $\frac{∂y}{∂t}\left(x,0\right)=0, 0<x<l.$
3. Solve the equation by the method of separation of variables.
4. An insulated rood of length $l$ has its ends A and B maintained at $0°$ and $100°$ respectively until steady state conditions prevail. If B is suddenly reduced to $0°$ and maintained at$ 0°$ find the temperature at a distance $x$ from A at time $t.$

**UNIT-IV**

**FOURIER TRANSFORMS**

**Part –A**

1. Define: Fourier Integral Theorem.
2. Define: Fourier Transform
3. State: Linear Property of Fourier Transformation.
4. Find the Fourier sine transform of *f* (*x*) = *e*-*x*
5. Find the Fourier cosine transform of *f* (*x*) = *e*-*x*
6. Define: Convolution of two functions
7. State: Convolution Theorem.
8. Define: Fourier Cosine transform.
9. State the Fourier transforms of the derivative of a function.
10. Define: Sine Fourier Integral.
11. State: Inverse Fourier Transform.
12. Define: Cosine Fourier Integral.
13. Write Inverse Fourier Cosine Transform?
14. State Parsevals Identity in Fourier Transforms.
15. State Parsevals Identity in Fourier Sine Transforms.
16. State Parsevals Identity in Fourier Cosine Transforms.
17. Define self Reciprocal.
18. State any two properties of Fourier Transforms.
19. State and prove change of scale Property.
20. Find the Fourier Sine Transform of f(x) = $\frac{1}{x}$
21. Find the Fourier Cosine Transform of f(x) = *eax*
22. Find the Fourier Sine Transform of f(x) = *e*-2x
23. Find the Fourier Cosine Transform of f(x) = *e*-2*x*
24. Define: Fourier sine transform and its inverse
25. Define: Fourier cosine transform and its inverse.

**PART –B**

1. If *f* (*x*) = $\left\{\begin{array}{c}1, \left|x\right|\leq 1\\0, \left|x\right|>1\end{array}\right.$ as a Fourier integral .Hence Evaluate 
2. Find the Fourier sine integral for $f (x) =e^{-βx}$, Hence show that 
3. Using Fourier cosine integral representation of an appropriate function,

 show that 

1. Find the Fourier transform of *f* (*x*) = $ \left\{\begin{array}{c}1, \left|x\right|<a\\0, \left|x\right|>a\end{array}\right.$
2. Find the Fourier cosine transform of *f* (*x*) = 
3. Find the Fourier cosine transform of *f* (*x*) = [5*e* -2*x*+2*e*-5*x*]
4. Find the Fourier sine transform of *f* (*x*) = [5*e*-2*x*+2*e*-5*x*]
5. Find the Fourier Sine Transform of *f* (*x*) = 2*e*-*x*
6. Find the Fourier Cosine Transform of *f* (*x*) = $\frac{e^{-ax}}{x}$
7. Find the Fourier sine transform of *f* (*x*) = 
8. Find the Fourier Sine Transform of *f* (*x*) = $\left\{ \begin{array}{c}x, 0<x<1\\2-x, 1<x<2\\0, x>2\end{array}\right.$
9. Findthe Fourier cosine and sine transform of  .
10. Findthe Fourier cosine and sine transform of 
11. Findthe Fourier cosine and sine transform of 
12. Find the Fourier cosine transform of [6e-4*x* +8e-2*x*]

 **PART C**

1. Find the Fourier Transform of *f* (*x*) = $\left\{\begin{array}{c}1-x^{2}, \left|x\right|\leq 1\\0, \left|x\right|>1\end{array}\right. $Hence Prove that 
2. Find the Fourier transform of  and hence deduce that 
3. Show that the Fourier Transform of *f* (*x*) = $\left\{\begin{array}{c}a^{2}-x^{2}, \left|x\right|<a \\ 0, \left|x\right|>a>0\end{array} \right. $is  .
4. Find the Fourier cosine transform of f(x)$ if f(x) =$ $\left\{\begin{array}{c}cosx, 0<x<a\\0, x>a\end{array}\right. $
5. Find the Fourier Transform of f(x)$=\left\{\begin{array}{c}1-\left|x\right|, \left|x\right|<1\\0, \left|x\right|>1\end{array}\right. $and hence deduce that 
6. Evaluate by transforms of 
7. Using Parsevals identity to calculate  if *a* > 0.
8. Using Parsevals identity to calculate if *a* > 0.
9. Find the Fourier Transform of *f* (*x*) = $\left\{\begin{array}{c}1 for \left|x\right|<2\\0 for \left|x\right|>2\end{array}\right. $Hence Prove that 
10. Find the Fourier transform of 

**UNIT-V**

**Z-TRANSFORMS**

**PART-A**

1. Define: Z-Transform
2. State: Initial value problem in Z Transform.
3. State: Linear Property in Z Transform.
4. State: Final Value theorem in Z Transform.
5. Write any two property of Z Transform.
6. Prove that Z[*an*] =$ \frac{z}{z-a}$
7. Prove that Z[1] =$ \frac{z}{z-1}$ .Find also the region of convergence.
8. Prove that Z[($-$1)*n*]=$ \frac{z}{z+1}$.
9. Find Z[$\frac{1}{n}$].
10. Find Z [$\frac{a^{n}}{n!}$]
11. Prove that 
12. Find 
13. Find 
14. Find Z [*an*2 + *bn* + *c*]
15. Find Z [(*n* + 1)(*n* + 2)]
16. Find Z [$e^{-iat}]$
17. Find Z[$e^{iat}]$
18. Find Z [8*n*2 + 4*n* + 9]
19. Find 
20. Find Z [(*n* + 3)(*n* + 5)]
21. Find 
22. Prove that Z [$\frac{1}{n!}$] = 
23. Find 
24. Find Z [2].
25. Find 

**PART-B**

1. Prove that Z[1)n] =$ \frac{z}{z+1}$. Find also the region of convergence.
2. Prove that 
3. Prove that 
4. Find ****
5. Prove that Z[*n an*] =$ \frac{az}{ (z-a)2}$
6. Find and 
7. Find Z(*sin at*)
8. Find Z(*cos at*)
9. Find (i) Z [*an*2 *+ bn + c*] (ii) Find Z[(*n+*1)(*n+*2)]
10. Find Z[*n*2]
11. Find the inverse Z-Transform of, by the method of partial fractions.
12. Find  and 
13. Find the initial and final values of the function 
14. Find 
15. Find 

**PART- C**

1. Find Z(*rn cosn*$θ$) and Z(*rn sin*$nθ)$
2. Solve:  using Z-Transform.
3. Find  and 
4. Find, by the method of partial fractions.
5. Find, by the residue theorem.
6. Find, by the method of partial fractions.
7. Find, by the method of partial fractions.
8. Find, by the method of partial fractions.
9. Solve: given  using Z -Transform
10. Solve: ,given  using Z –Transform.